BOUND STATE SOLITON DESCRIPTION OF LOW PARTIAL WAVE OCTET BARYON RESONANCES

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ABSTRACT

A version of the bound state soliton model which allows both η and K bound states is used to study low partial wave octet baryon resonances. It is found that negative parity S–wave resonances are well described within this framework. A possible interpretation of the P–wave resonances is also discussed.

Although the idea of describing some of the nucleon resonances as eta-nucleon bound state was proposed long ago [1], only recently this possibility was investigated in the context of the soliton models. In Ref.[2] it was shown that when an alternative fourth order term is included together with the usual Skyrme term, an S–wave eta-soliton bound state appears. This eta-soliton bound state has been identified with the N(1535) S–wave resonance, which has a large branching ratio for decays into ηN. The introduction of the alternative fourth order term is justified in the framework of chiral perturbation theory [3]. Moreover, some constraints on the strength of such a term with respect to the one of the Skyrme term can be obtained from phenomenological sources [4]. The validity of this scheme has been tested by the calculation of the eta photoproduction amplitudes from nucleons [5]. The purpose of the present article is to extend this picture to the description of those strange baryon resonances which have a relative large branching ratio for the decay into the corresponding low lying hyperon state and an η. As in Ref.[2] our work is based on the bound state soliton approach proposed by Callan and Klebanov [6], which has been shown to be very successful in the description of hyperon properties [7, 8].

We start with the effective soliton Lagrangian with an appropriate symmetry breaking term, expressed in terms of the $SU(3)$–valued chiral field $U(x)$ as

$$\Gamma = \Gamma^{(2)} + \Gamma^{(4)} + \Gamma_{WZ} + \Gamma_{SB}. \tag{1}$$

$\Gamma^{(2)}$ is the usual kinetic term

$$\Gamma^{(2)} = -\frac{f_\pi^2}{4} \int d^4x \ \mathrm{Tr}(L_\mu L^\mu) \tag{2}$$

while $\Gamma^{(4)}$ is the fourth order interaction term written as

$$\Gamma^{(4)} = \int d^4x \left\{ \frac{x}{32e^2} \mathrm{Tr}[L_\mu, L_\nu]^2 + \frac{1-x}{16e^2} \{ (\mathrm{Tr} L_\mu L_\nu)^2 - (\mathrm{Tr} L_\mu L^\mu)^2 \} \right\}, \tag{3}$$

where $L_\mu = U^\dagger \partial_\mu U$, U being the chiral soliton field.

The first term in RHS of Eq.(3) is the standard quartic Skyrme term (e is the so–called Skyrme parameter), that yields no interaction between the soliton and the η field. The second contribution is the "alternative" quartic term introduced in Ref.[2]. The parameter x weights the relative strength of these two terms, its range going from 0 to 1.

In Eq.(1) Γ_{SB} is responsible for the explicit breaking of chiral symmetry. We use the following form for Γ_{SB}

$$\Gamma_{SB} = \int d^4x \left\{ \frac{f_\pi^2 m_\pi^2 + 2f_K^2 m_K^2}{12} \mathrm{Tr} \left[U + U^\dagger - 2 \right] \right.$$

1

$$+\frac{f_\pi^2 m_\pi^2 - f_K^2 m_K^2}{6} \, \mathrm{Tr}\left[\sqrt{3}\lambda^8 \left(U + U^\dagger\right)\right]$$
$$\left. -\frac{f_K^2 - f_\pi^2}{12} \, \mathrm{Tr}\left[\left(1 - \sqrt{3}\lambda^8\right)\left(U\partial_\mu U^\dagger \partial^\mu U + U^\dagger \partial_\mu U \partial^\mu U^\dagger\right)\right]\right\} , \qquad (4)$$

where λ^8 is the eighth Gell-Mann matrix, f_π is the pion decay constant ($= 93$ MeV empirically), f_K is the kaon decay constant and m_π and m_K represent the pion and kaon masses respectively. Eq.(4) accounts for both the finite mass of the pseudoscalar mesons and the empirical difference between their decay constants. Finally, Γ_{WZ} is the non–local Wess–Zumino action

$$\Gamma_{WZ} = -i\frac{N_c}{240\pi^2}\int \, \mathrm{d}^5x \; \varepsilon^{\mu\nu\alpha\beta\gamma} \; \mathrm{Tr}(L_\mu L_\nu L_\alpha L_\beta L_\gamma) . \qquad (5)$$

In order to describe the eta-kaon-soliton system we introduce a generalized Callan-Klebanov ansatz for the chiral field given by

$$U = \sqrt{U_\pi}U_K U_\eta \sqrt{U_\pi} , \qquad (6)$$

where

$$U_K = \exp\left[i\frac{\sqrt{2}}{f_K}\begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix}\right] , \qquad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} , \qquad (7)$$

and

$$U_\eta = \exp\left[i\frac{\eta\lambda_8}{f_\eta}\right] . \qquad (8)$$

Finally, U_π is the soliton background field written as a direct extension to $SU(3)$ of the $SU(2)$ field u_π, i.e.,

$$U_\pi = \begin{pmatrix} u_\pi & 0 \\ 0 & 1 \end{pmatrix} . \qquad (9)$$

It should be noticed that the η field has been introduced as the eight component of the pseudoscalar octet. A similar approximation has been used in previous calculation [2, 5]. Considering the rather small mixing angle in the physical η this appears as a reasonable approximation.

As mentioned above, our effective action includes chiral symmetry breaking effects that lead to different decay constants for the different pseudoscalar mesons. In particular, f_η is related to the pion and kaon decay constants by

$$f_\eta^2 = \frac{4}{3}f_K^2 - \frac{1}{3}f_\pi^2 . \qquad (10)$$

Using the empirical ratio $\chi_K = f_K/f_\pi = 1.22$, one gets $f_\eta/f_\pi = 1.28$. This value agrees well with the estimation given in Ref.[9]. Within the present model the η mass is given in terms of the pion and kaon masses by

$$m_\eta^2 = \frac{4\chi_K^2 m_K^2 - m_\pi^2}{4\chi_K^2 - 1}. \tag{11}$$

One can easily see that for $\chi_K \neq 1$ there are corrections to Gell-Mann–Okubo mass formula. Using the empirical values of χ_K, m_π and m_K we obtain $m_\eta = 539$ MeV to be compared with the empirical value $m_\eta^{emp} = 549$ MeV

Following the usual steps of the bound state approach we expand up to second order in the kaon and eta field. To this order, these fields are decoupled one from the other, their interactions being only with the soliton background field. Therefore, the kaon-soliton effective action reduces to the one discussed in Ref.[4]. One the other hand, the resulting eta-soliton effective action has some differences with respect to the one used in Ref.[2]. This is due to the more realistic symmetry breaking action used in the present calculation. Using the hedgehog ansatz for the $SU(2)$ soliton field

$$u_\pi = \exp\left[i\,\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\,F(r)\right] \tag{12}$$

our eta-soliton action to $\mathcal{O}(N_c^0)$ reads

$$L_{\eta-sol} = \frac{1}{2}\int d^3x \left\{\alpha\dot\eta^2 - \left[\rho(\boldsymbol{\nabla}\eta)^2 - (\beta-\rho)(\partial_r\eta)^2\right] + \gamma^2\eta^2\right\}, \tag{13}$$

where

$$\alpha = 1 + \frac{1-x}{e^2 f_\eta^2}\left[F'^2 + 2\frac{\sin^2 F}{r^2}\right], \tag{14}$$

$$\beta = 1 + \frac{1-x}{e^2 f_\eta^2}\,2\frac{\sin^2 F}{r^2}, \tag{15}$$

$$\rho = 1 + \frac{1-x}{e^2 f_\eta^2}\left(F'^2 + \frac{\sin^2 F}{r^2}\right), \tag{16}$$

$$\gamma^2 = m_\eta^2 - \frac{m_\pi^2 f_\pi^2}{3 f_\eta^2}\left(1 - \cos F\right). \tag{17}$$

Using the standard partial-wave decomposition of the η field we find that the eingenmodes that diagonalize the hamiltonian in each partial wave can be written as

$$\eta(\boldsymbol{r}, t) = e^{-i\omega_\eta t}\,\eta(r)\,Y_{lm}(\theta, \phi), \tag{18}$$

where $\eta(r)$ satisfies the eigenvalue equation

$$\left\{\frac{1}{r^2}\partial_r\left[r^2\beta\partial_r\right] + \alpha\,\omega_\eta^2 - \frac{l(l+1)}{r^2}\rho - \gamma^2\right\}\eta = 0. \tag{19}$$

3

In our numerical calculations we use two sets of values for the parameters appearing in the effective lagrangian. In SET A we keep f_π, f_K and m_π fixed to their empirical values. SET B corresponds to the case often used in skyrmion physics in which the pion is considered as massless and f_π takes the value $f_\pi = 64.5$ MeV. In the latter case, the ratio f_K/f_π is taken to its empirical value ($= 1.22$). In both cases the value of e is adjusted to reproduce the empirical $\Delta - N$ mass difference and m_K is taken to the empirical value $m_K = 495$ MeV. As already mentioned the kaon sector of the present model has been studied in Ref.[4]. In particular, the kaon binding energies ω_K and hyperfine splitting constants c_K have been calculated for different values of the mixing parameter x. It was found that the hyperon masses are better reproduced when the values $x = 0.33$ (for SET A) and $x = 0.66$ (for SET B) are used.

Using the parameter sets given above we have solved the η eigenvalue equation Eq.(19). As in previous calculations [2, 5] we have found that the only bound state appears in the S–wave. The corresponding eigenenergy ω_η as a function of x for both SET A and SET B is shown in Fig.1. When comparing our results with those of Ref.[2] we observe that the eta is less bound in our calculation. This is mainly due to the incorporation of symmetry breaking terms that take into account the difference between the η decay constant and the pion one. A similar effect was found in the kaon sector [7]. Using the values of x that lead to the best agreement with empirical ground state hyperon masses we find $\omega_\eta(x = 0.33) = 467$ MeV (SET A) and $\omega_\eta(x = 0.66) = 517$ MeV (SET B).

In order to obtain the correct baryon quantum numbers we have to consider the $SU(2)$ soliton isospin rotations. This leads to the hyperfine corrections to the masses. Since the eta is bound in an S–wave the corresponding hyperfine splitting constant vanishes to $\mathcal{O}(N_c^{-1})$. Therefore, the mass formula for spin 1/2 particles reads:

$$M = M_{sol} + n_K \omega_K + n_\eta \omega_\eta + \frac{1}{2\mathcal{I}} \left[(1 - c_K)I(I + 1) + \frac{3}{4}c_K^2 \right], \qquad (20)$$

where M_{sol}, $\mathcal{I}$ and I are respectively the soliton mass, moment of inertia and isospin. ω_K and c_K are the kaon energy and hyperfine constant, n_K and n_η represent the number of bound kaon(s) and η meson(s), related to the strangeness and parity quantum numbers, respectively.

In Table 1 we list the calculated masses of the 1/2 baryon octet together with the corresponding S–wave negative parity resonances. Also shown are the empirical values taken from Ref.[10]. In the case of the $\Xi(1950)$ the assignment is only tentative since the spin and parity of this state have not been empirically determined yet. The values of ω_K and c_K used to construct this table have been taken from Ref.[4].

Apart from the negative parity S–wave resonances there are some other low-

4

partial wave baryon resonances which also have a rather large η-baryon branching ratio. An example of such states is the P_{13} N(1710) resonance. To investigate whether these states could be described within the present model we have solved the η-eigenvalue equation, Eq.(19), in the continuum for different partial waves. The corresponding phase shifts as a function of the eta energy ω_η are shown in Fig.2. We plot the phase shifts for the lower partial waves for both sets of parameters. As we see, the $l = 0$ waves show the typical behaviour due to the presence of a bound state. On the other hand, although the interaction is attractive in all the other partial waves, there is no resonance even for $l = 1$. Similar results are obtained for other values of the x-parameter. At this point it is important to recall that, within the bound state model, most of the hyperon resonances with $l \geq 2$ can be understood as kaon-soliton resonance states [11].

The results of Table 1 show that a comprehensive description of low–lying S–wave baryon resonances in terms of eta-kaon-soliton bound states is reliable. The presence of a bound η meson accounts fairly well for the mass difference between negative parity states and the corresponding ground state baryons. However the predicted masses are always too small by 50–100 MeV, the difference with the experimental values being larger with SET A parameters. The qualitative agreement could be partially improved by choosing a value of x closer to 1, at the price of a worse description of the hyperfine splitting. The problem, in any case, cannot be solved within this simple model approximations since they implicitly imply the constraint $\omega_\eta < m_\eta$ which is not satisfied phenomenologically (for instance, $m_{N(1535)} - m_N > m_\eta$). In order to obtain a better quantitative agreement with the experimental mass spectrum one has to consider the coupling with the pion vibrational modes. This "coupled channels" procedure has already provided a remarkable improvement in the predictions of the η-photoproduction observables [12]. It is also clear that this new available channel will have some influence on the P-wave phase shifts. Whether this is enough to produce some resonance behaviour in such a channel is however an open question. Work along these lines is in progress.

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Table and Figure Captions

Table 1 : Calculated excitation energies (in MeV) taken with respect to the nucleon mass as compared to the experimental ones. SET A corresponds to $f_\pi = 93$ MeV, $e = 4.26$ and $m_\pi = 138$ MeV while SET B corresponds to $f_\pi = 64.5$ MeV, $e = 5.45$ and $m_\pi = 0$. In both cases, $f_K/f_\pi = 1.22$ and $m_K = 495$ MeV. For SET A x is taken to the value $x = 0.33$ while for SET B to $x = 0.66$.

Fig. 1 : The η bound state energy as a function of the x parameter. The values of the other parameters are as in Table 1.

Fig. 2 : Eta-soliton phase shifts corresponding to partial waves with $l \leq 3$ as a function of the η energy. The values of the parameters are as in Table 1.

Table 1

n_K	n_η	I	J^P	SET A	SET B	Emp.	Particle
0	1	1/2	$1/2^-$	467	517	596	N(1535)
1	0	0	$1/2^+$	172	165	177	Λ
1	1	0	$1/2^-$	639	682	731	$\Lambda(1670)$
1	0	1	$1/2^+$	252	243	254	Σ
1	1	1	$1/2^-$	719	760	811	$\Sigma(1750)$
2	0	1/2	$1/2^+$	393	377	379	Ξ
2	1	1/2	$1/2^-$	860	894	1011	$\Xi(1950)$?